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Cost Minimization Assignment Problem Using Fuzzy Quantifier

G. Nirmala[1] and R. Anju[2]

[1]Associate Professor of Mathematics, PG & Research, Dept. Of Mathematics, K.N. Govt. Arts College for Women (Autonomous) Thanjavur-613007, Tamil Nadu, India.

[2]Lecturer in Mathematics, Bharathidasan University Constituent College (W), Orathanadu, Thanjavur, Tamil Nadu, India .

Abstract: This work presents an assignment problem with the aid of triangular or trapezoidal fuzzy numbers. For finding the initial solution we have preferred the fuzzy quantifier and ranking method [6], also a method named ASM- Method [7] is applied for finding an optimal solution for assignment problem. This method requires least iterations to reach optimality, compared to the existing methods available in the literature. Here numerical examples are solved to check the validity of the proposed method.

Keywords: Assignment problem, Fuzzy quantifier, Ranking function, ASM method, optimal solution.

I. INTRODUCTION

Assignment Problem (AP) is a well-known topic and is used worldwide in solving real world problems. An Assignment Problem plays an important role in industry and other applications. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. In this problem C_{ij} denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one person can be assigned exactly one job, also each person can do atmost one job. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimum or the total profit is maximum.

In this work we investigate a more realistic problem, namely the assignment problem with fuzzy costs or times $\tilde{C}_{ij}$ represented by fuzzy quantifier which are replaced by triangular or trapezoidal fuzzy numbers. Since the objectives are to minimize the total cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number. First, to rank the objective values of the objective function by fuzzy ranking method [6].

A method named ASM-Method [7] is proposed to find the fuzzy optimal solution of fuzzy assignment problems by representing all the parameters as fuzzy quantifiers which are replaced by triangular fuzzy numbers. The advantages of the proposed method are also discussed. To illustrate the proposed method a fuzzy assignment problem is solved and the results obtained are also discussed. The proposed method is easy to understand and to apply for finding an optimal solution of assignment problem occurring in real life situation.

II. PRELIMINARIES

A. *Basic definitions*

Definition A1: The characteristic function $\mu_{\tilde{A}}$ of a crisp set A⊆ X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value
assigned to the element of the universal set fall within specified range. i.e., $\mu_{\tilde{A}}: X \longrightarrow [0,1]$. The assigned value indicate the membership grade of the element in the set. The function $\mu_{\tilde{A}}(x)$ is called membership function and the set

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x): x \in X)\}$$

defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition A2: A fuzzy set $\tilde{A}$, defined on universal set of real numbers, is said to be a fuzzy number if its membership function has the following characteristics:

(i) is convex. i.e.,$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq minimum(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))\ \forall\ x_1, x_2 \in \tilde{A}\ \forall\ \lambda \in [0,1]$.

(ii) is normal. i.e., $\exists\ x_0 \in \tilde{A}$ such that $\mu_{\tilde{A}}(x_0) = 1$.

(iii) $\mu_{\tilde{A}}$ is piecewise continuous.

Definition A3: A fuzzy number $\tilde{A}$ = (a,b,c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & x = b \\ \frac{(x-c)}{(b-c)}, & b \leq x \leq c \end{cases}$$

where $a, b, c \in R$

Definition A4: A fuzzy number $\tilde{A}$ = (a,b,c,d) is called a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{(x-d)}{(c-d)}, & c \le x \le d \end{cases}$$

where $a, b, c, d \in R$

III. Arithmetic Operations Between Two Triangular And Trapezoidal Fuzzy Numbers

In this subsection, the arithmetic operations, required for the proposed algorithm, are reviewed (Kaufmann & Gupta, 1985).

Let $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers then

(1) $A \oplus B = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2)$
(2) $A \ominus B = (a_1 - c_2,\ b_1 - b_2,\ c_1 - a_2)$

Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers then

(1) $A \oplus B = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2,\ d_1 + d_2)$
(2) $A \ominus B = (a_1 - d_2,\ b_1 - c_2,\ c_1 - b_2,\ d_1 - a_2)$

IV. Ranking Function

A convenient method for comparing of fuzzy number is by use of ranking function (Liou & Wang, 1992).

A ranking function $\mathfrak{R} : F(R) \longrightarrow R$, where $F(R)$ is set of all fuzzy numbers defined on set of real numbers, maps each fuzzy number into a real number.

For a triangular fuzzy number $\tilde{A} = (a, b, c)$, $\mathfrak{R}$ is given by

$$\mathfrak{R}(A) = \frac{a+2b+c}{4}$$

For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, $\mathfrak{R}$ is given by

$$\mathfrak{R}(A) = \frac{a+b+c+d}{4}$$

V. Fuzzy Quantifier

Fuzzy quantifiers are fuzzy numbers that take part in fuzzy propositions. Fuzzy quantifiers that characterize linguistic terms such as about 10, much more than 100, atleast 5 of the first kind and almost all, about half, most and so on of the second kind.

One of them is the form

p: There are Qi's in I such that $\nu(i)$ is F,

where $\nu(i)$ of ν is a variable, Q is a fuzzy number expressing linguistic term, F is a fuzzy set of variable ν.

Example:

p: "There are about 10 students in a given class whose fluency in English is high".

Here Given a set of students, I, the value of $\nu(i)$ of variable ν

represents the degree of fluency in English of student I, F is a fuzzy set that expresses the linguistic term high, and Q is a fuzzy number expressing the linguistic term about 10.

VI. ASM Method

Step 1: Construct the Assignment Problem.

Step 2: Subtract each row entries of the assignment table from the respective row minimum and then subtract each column entries of the resulting assignment table from respective column minimum.

Step 3: Now there will be atleast one zero in each row and in each column in the reduced cost matrix. Select the first zero (row-wise) occurring in the cost matrix. Suppose $(i,j)^{th}$ zero is selected. Count the total number of zeros (excluding the selected one) in the i^{th} row and j^{th} column. Now select the next zero and count the total number of zeros in the corresponding row and column in the same manner. Continue it for all zeros in the cost matrix.

Step 4: Now choose a zero for which the number of zeros counted in step 3 is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in step 3 then choose a $(k,l)^{th}$ zero breaking tie such that the total sum of all the elements in the k^{th} row and l^{th} column is maximum. Allocate maximum possible amount to that cell.

Step 5: After performing step 4, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.

Step 6: Check whether the resultant matrix possesses at least one zero in each row and in each column. If not, repeat step 2, otherwise go to step 7.

Step 7: Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

The cost matrix $[c_{ij}]_{nxn}$ for the proposed method is given in the following table I.

TABLE I

Jobs → / Persons ↓	1	2	3	…j…	n
1	c_{11}	c_{12}	c_{13}	..c_{1j}..	c_{1n}
2	c_{21}	c_{22}	c_{23}	..c_{2j}..	c_{2n}
-	-	-	-	-	-
i	c_{i1}	c_{i2}	c_{i3}	..c_{ij}..	c_{in}
-					
n	c_{n1}	c_{n2}	c_{n3}	..c_{nj}..	c_{nn}

VI. NUMERICAL EXAMPLE – COST MINIMIZATION ASSIGNMENT PROBLEM

Consider the following cost minimization Assignment Problem. Here the cost $(\tilde{C}_{ij})$ involved in executing a given job is considered as fuzzy quantifiers which characterize the linguistic variables are replaced by triangular fuzzy numbers. The problem is then solved by proposed method to find an optimal solution.

$$\tilde{C}_{ij} = \begin{bmatrix} \textit{Reasonably high} & \textit{drastically low} & \textit{Reasonably low} & \textit{Reasonably high} \\ \textit{High} & \textit{ok} & \textit{ok} & \textit{Reasonably low} \\ \textit{Medium} & \textit{Reasonably high} & \textit{Low} & \textit{Medium} \\ \textit{drastically high} & \textit{Reasonable} & \textit{drastically low} & \textit{High} \end{bmatrix}$$

Solution

The linguistic variables showing the qualitative data is converted into quantitative data using the following table. The linguistic variables are represented by triangular fuzzy numbers.

TABLE II

OK	(8,10,12)
Drastically high	(16,18,20)
Reasonable	(18,20,22)
High	(17,19,21)
Reasonably high	(5,10,15)
Medium	(12,14,16)
Reasonably low	(9,11,13)
Low	(6,8,10)
Drastically low	(7,9,11)

Now from table 1, we have

$$\tilde{C}_{ij} = \begin{bmatrix} (5,10,15) & (7,9,11) & (9,11,13) & (5,10,15) \\ (17,19,21) & (8,10,12) & (8,10,12) & (9,11,13) \\ (12,14,16) & (5,10,15) & (6,8,10) & (12,14,16) \\ (16,18,20) & (18,20,22) & (7,9,11) & (17,19,21) \end{bmatrix}$$

Now, by using the ranking technique the above AP is

$$\begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{array} \begin{array}{c} \begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} \\ \begin{pmatrix} 10 & 9 & 11 & 10 \\ 19 & 10 & 10 & 11 \\ 14 & 10 & 8 & 14 \\ 18 & 20 & 9 & 19 \end{pmatrix} \end{array}$$

After applying ASM-Method, the optimal solution is

$B_1 \longrightarrow J_1, B_2 \longrightarrow J_4, B_3 \longrightarrow J_2, B_4 \longrightarrow J_3$ and

the minimum assignment cost = Rs. (9 + 11 + 10 + 10)=Rs.40

VI. CONCLUSION

In this work, the assignment costs are taken as fuzzy quantifier that characterize linguistic variables are termed as triangular fuzzy numbers which are more realistic and general in nature. Moreover, by the application of ranking fuzzy number the fuzzy assignment problem has been transformed into crisp assignment problem and then obtained the optimal solution using ASM method. Numerical example shows that the total cost obtained is optimal. Thus it can be concluded that ASM-Method provides an optimal solution directly, in fewer iterations, for the assignment problems. As this method consumes less time and is very easy to understand and apply, so it will be very helpful for decision makers who are dealing with logistic and supply chain problems.

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